The study on self-sufficient catastrophe bailout programs

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Abstract

This study shows the feasibility that a natural catastrophe insurance fund (NCIF) may achieve financial self-sufficiency by employing three bailout programs, including pre-funding, loan-financing and equity-financing, to support the insurer during bad years. In the three bailout programs, different accounting procedures of insurer and NCIF are illustrated, and the loss sequence is calibrated by using global catastrophic insured losses. Based on these accounting procedures, the 30-year cash flows of insurer and NCIF are simulated under such loss model with reasonable parameter values. The results of numerical analysis indicate that three bailout programs can balance the financial revenue and expenditure of NCIF in the long term, and this paper also implies that the authority can develop the related NCIF scheme to smoothen the impact of catastrophe risk in the insurance industry.

Keywords: natural catastrophe insurance fund, pre-funding bailout program, loan-financing bailout program, equity-financing bailout program

1. Introduction

Natural catastrophes (NatCat) have become more frequent over the past three decades. A country needs an insurance mechanism in response to the economic consequences of NatCats. Kunreuther (1996) suggested a disaster-management insurance program to minimize losses from infrastructure damage. It was also suggested that a new form of reinsurance coverage against catastrophic losses from natural disasters should be created to protect insurers against potential insolvency. Dacorogna et al. (2013) pointed out that according to US-GAAP and new IFRS, insurers are forbidden to carry equalization reserves over future business if no loss occurred. This might weaken the insurers' capability to respond to NatCat. Froot (2001) examined the clinical data in the US and proposed eight explanations in order to understand why prices of reinsurance are so high while demand for reinsurance is so low. A catastrophe insurance fund brings a different trade-off between risk and return than the insurance product offered in the private market, and it can increase efficiency in the private market (Boulatov and Dieckmann 2013). Most governments have allocated substantial resources for the improvement of preventing and forecasting NatCats. Cummins (1988) also suggested that guaranty funds with flat premiums create adverse incentives in insurance markets. Therefore, it would be more prudent to set up a national catastrophe insurance fund (NCIF) [1] (or a scheme with similar functions) to handle large claims or losses caused by NatCats. For example, Boulatov and Dieckmann (2013) illustrated that the involvement of disaster insurance funds and the well-designed insurance in the NatCat market could increase the demand in the private market.

In the real world, there are many similar funds: The National Flood Insurance Program and the Hawaii Hurricane Relief Fund in the US; the Natural Disaster Fund in New Zealand; the National Catastrophe Insurance Fund in Thailand; PT. Asuransi MAIPARK in Indonesia; the Earthquake Insurance Pool in the Philippines. Generally, NCIF and similar schemes offer some advantages in minimizing NatCat risk. First, they can protect the policyholders by ensuring the affordability and the availability of the insurer. Second, NatCat risk can be diversified to insureds, insurers and reinsurers since NCIF would offer insurers different types of bailouts to cover large claims. The insurers can be allowed to pay back by instalment or

[1] In this study, the national catastrophe insurance fund (NCIF) is set as a non-profit-oriented organization.

new-issued equity. Thus, NCIF could be seen as a mechanism to encourage insurance industry to regain normal operations after NatCat. Third, NCIF ensures capacity to insurance companies at a reasonable price in different ways. However, although NCIF offers benefits, governments cannot always cover NCIF after each disaster. Covering the aftermath of every large NatCat may weaken a country's fiscal position if governments constantly have to pay for reconstruction, disrupting their sources of revenue (Keen et al. 2003).

From the perspective of NatCat risk, the claim is below the expectation in most years. However, the actual claim size may easily exceed the expected claim one so much in the large NatCats years, as the annual premium is insufficient to cover the sudden liabilities. In the subsequent years after a large NatCat, the insurance market becomes highly unstable. The premium adjustment might be enough to compensate the insurer's losses in the long-run, assuming the insurers can even survive from the financial distress. Originally, the insurer's risk-adjusted capital with high quantile is applied to cover possible and unexpected large claims. However, after paying large position of claims the capital may not be sufficient for paying new coming claims, or the remaining capital is insufficient for the capital requirement to write the new policies. As a result, the cost of raising capital will be high and there may be less cash available when an insurer is in distress. Under this circumstance, the insurer would be forced to quit the insurance market. For the purpose of creating a more comprehensive insurance market, NCIF should play a critical role softening the peak impact of NatCat risk in the insurance market whenever the insurance market scheme becomes unbalanced.

To reduce the impact of natural catastrophes on fiscal distribution, NCIFs have to achieve long-term financial self-sufficiency. That is, they can operate by themselves without government support for a long time even if they may suffer from large NatCat claims during bad years. To our best knowledge, few literatures study the balance between NCIF's financial revenue and its own expenditure. This study offers three programs, including pre-funding, loan-financing and equity-financing, to balance NCIF's budget for the long term. Also, the accounting procedures of the three programs will be demonstrated in detail to present their feasibilities.

First, the pre-funding program means that NCIF gathers annual premiums, and then provides a certain bailout to insurers in bad years. The premium rate in this program is associated with the probability of future possible large claims that insurers cannot afford. The concept is similar to catastrophe reserves which are ready for large claims. The function serves as an insurance for private insurance companies. Most NCIFs adopt methods against NatCat risk. Since NCIF is a non-profit organization and the government provides its original funds, it has lower capital cost than reinsurance market. The premium charged by NCIF could help NCIF to achieve long-term financial balance. The insurers receiving the bailout do not have to pay back. However, the disadvantage emerging from the model is that the program may lead to moral hazard since the insurers would not like to do their best in reducing claims. In this function, premium rates required by NCIF will be the key issue. Unreasonable low premium rates will cause financial shortfall, but higher rates seem to discourage insureds, insurers and reinsurers from join this program.

The second program is called loan-financing meaning that NCIF offers the insurer a standby loan. The insurer needs to pay back the loan first when his profit after tax becomes positive. The NCIF serves as the bank offering the insurer a large position of credit. The loan rate charged by NCIF should be lower than the premium rate of pre-funding program because the loan needs to be paid back. If an insurer can survive from lager claims, he could get profit and then pay for the loan as the market condition becomes better. The key point of the feasibility of such a program is that large NatCats occur cyclically. The findings exhibit the cyclical characteristics of insured losses data, and the numerical analysis ensures the loan financing program is effective in allowing NCIF to achieve self-sufficiency in the long term.

The last program that can be used by NCIF is called equity-financing, where the insurer mired in difficulties from financial distress exchanges bailout from NCIF by issuing new equity. The NCIF owing insurer's equity will sell these shares back once insurer's profit after tax becomes positive. A certain proportion of the income is reserved for the next bailout. Similarly, if the market condition becomes good eventually, an insurer can get positive profit after tax in sequent years. Intuitionally, this could make share prices rise. The NCIF can obtain capital benefit by selling shares. The key point for this program to work effectively depends on whether large NatCats occur cyclically.

To the best of our knowledge, the proposed concepts on bailout programs to balance the long-term financial budget of NCIF are new in related literature. It is certain that the proposed bailout program could lead NCIF to achieve financial self-sufficiency in the long term. Thus, NCIF's operation will be less affected by government budget and policies. These three programs are promising for several reasons. First, the pattern of global insured losses exhibits the cyclical characteristics of NatCat events which is quite suitable for the proposed programs. The time diversification for peak claims would be effective in smoothening NatCat risk. Second, the proposed accounting procedures of NCIF and the insurer are simulated their future cash flows over 30 years. The effects of NCIF's different bailout programs for insurers would post diversified performance such as operating result, liability and profit before tax. Also, those effects would contribute positively to NCIF's financial revenue and expenditure. Third, the results of numerical simulation based on normal parameter assumption present the feasibility of financial self-sufficiency if NCIF adopts these bailout programs in this study.

The rest of the paper is organized as follows: Section 2 develops the loss model, the technical premium, accounting procedures for NCIF and insurer using one of the three programs (pre-funding, loan financing or equity financing). Section 3 carries out the empirical experiment and numerical simulation. Then, the conclusion will be made in section 4.

2. The Model

For writing policies against NatCat risk, the insurer needs sufficient risk-adjusted capital (*RAC*) with high quantile estimated by loss model and the quantile (θ) corresponds to the occurrence probability (θ) of claim size. The insurer can raise liability to satisfy the shortfall of *RAC* if necessary, or take the bailout program offered by NCIF if the insurer falls into distress. Once the accumulated liability reaches the bailout trigger with a predetermined ratio (ρ), i.e. $\rho * RAC$, ρ, NCIF will save the insurer by adopting one of following three bailout programs. The corresponding accounting procedures of NCIF and insurer will be demonstrated in the following subsections. The cash flows between the NCIF and the insurer will be presented based on different bailout programs under the loss model. Furthermore, the calibrated loss model will be displayed as well.

2.1 The Loss Model

Assume $X(t)$ is the insured loss in year t, which is a sequence of independent and identically distributed random variables with generic random variable X. The risk-adjusted capital is defined as the Value-at-Risk $VaR[X;\theta]$ with the quantile, θ.

$$RAC(t) = F_X^{-1}[X(t)|X(t-1),\ \theta], \tag{1}$$

where F_X^{-1} is the inverse of the cumulative distribution function of X and θ means the risk cover ratio. $RAC(t)$ is treated as solvency capital requirement. At the beginning, the insurers must have the risk-adjusted capital ($RAC(1)$) for issuing NatCat policies in year 1. In an extreme case, the maximum claim to the insurer is $RAC(1)$. When the capital level is lower than $RAC(1)$ due to the claim loss, the insurers need to raise capital again to maintain the risk-adjusted capital level. Otherwise they will not be allowed to issue new policies again.

2.2 The premium

The technical premium ($TP(t)$) [2] for the year t should be a summation covering four components. They are the expected claims ($E[X(t)|X(t-1)]$), the adjustment of unexpected previous-year claims, risk premium of $RAC(t)$, and the internal expense and operational cost. The risk premium required by shareholders is usually regarded as the cost of capital ($\kappa \cdot RAC(t)$) where κ denotes the cost rate of capital. Certainly, $\kappa \cdot RAC(t)$ could be interpreted as the risk premium required by shareholders of the insurance company. Let $e(t)$ be the sum of internal expense and operational cost and τ denotes the tax rate. Thus, the technical premium $TP(t)$ (collected at time t-1) needs to satisfy an equilibrium equation between expected cash inflow and expected cash outflow as the following equation:

[2] Technical Premium is defined as an indicated risk premium calculated directly from a specific pricing model excluding any influence of credit rating or other judgment modification.

$$TP(t)(1+r)(1-\tau) = (E[X(t)] + X(t-1) - E[X(t-1)] + e(t))(1-\tau) + \kappa \cdot RAC(X) \cdot r \cdot \tau, \quad (2)$$

where r is the risk-free interest rate. The premium is also calculated by the following equation:

$$TP(t) = \frac{E[X(t)] + X(t-1) - E[X(t-1)] + e(t)}{1+r} + \frac{\kappa \cdot RAC(X) \cdot \tau}{(1+r)(1-\tau)}. \quad (3)$$

2.3. The basic accounting procedure of insurers

Assume an insurer has initial asset $RAC(1)$ for issuing NatCat policies in year 1, and he continues preparing $RAC(t)$ at the end of year *t*-1 for issuing policies in year *t*. The insurer is assumed to operate the business as well as he can. When the operational result is positive, the debt has the first priority to be paid and then the remaining will be distributed to shareholders as dividend. The insurer can raise some liability to match the minimum capital requirement for the new policies next year.

The new capital for next year is also set at the end of the year *t* when the actual total payment of the claims is known. The underwriting result $UR(t)$ at the end of year *t* is the result of premium income minus claim and expense as shown in equation (4),

$$UR(t) = TP(t) - X(t) - e(t). \quad (4)$$

Moreover, the insurer has two additional incomes: interest from previous-year capital and premiums, and the adjustment of risk-adjusted capital between this year and next year. Therefore, the operating result at the end of year *t* is

$$OR(t) = UR(t) + r \cdot (RAC(t) + TP(t)) + RAC(t) - RAC(t+1). \quad (5)$$

If the market is extremely severe and operating result of the insurer is less than negative risk-adjusted capital, i.e. $OR(t) < -RAC(t)$, the insurer defaults. Otherwise, the insurer continues to write policies. The cumulative liability at the end of year *t*, $L(t)$, consists of the previous-year's liability $L(t-1)$, the increment of risk-adjusted capital $RAC(t+1) - RAC(t)$ and the operating result $OR(t)$. It could be modelled by

$$L(t) = \max\left[L(t-1) + RAC(t+1) - RAC(t) - OR(t),\ 0\right]. \quad (6)$$

If the cumulative liability drops, it means that the positive operating result has been used in paying the liability. The adjusted operating result at the end of year *t*, $AOR(t)$ is estimated by:

$$AOR(t) = OR(t) - \max\left[L(t-1) - L(t),\ 0\right]. \tag{7}$$

So, profit before taxes $PBT(t)$ could be obtained by using adjusted operating result minus the cost of liability $cL(t-1)$ as shown in equation (8)

$$PBT(t) = AOR(t) - cL(t-1), \tag{8}$$

where c is the interest rate of the liability. If the profit before taxes is negative, the government provides a tax shield with rate γ to encourage the insurance market by increasing future capital. The cumulative deferred tax $DTAX(t)$ is calculated by

$$DTAX(t) = \max\left[DTAX(t-1) - \gamma \cdot PBT(t),\ 0\right]. \tag{9}$$

In contrast, if the profit before taxes is positive, the taxable amount of the profit should be $PBT(t)$ excluding $DTAX(t-1)$. $TAX(t)$ is charged based on the tax rate, τ. Therefore, the total taxes could be derived via equation (10)

$$TAX(t) = \tau \max\left[PBT(t) - DTAX(t-1),\ 0\right]. \tag{10}$$

And the profit after taxes at the end of year t is

$$PAT(t) = PBT(t) - TAX(t). \tag{11}$$

2.4 The accounting procedure for insurers and NCIF with pre-funding

In the first case, we assumed that the NCIF charges the insurer a pre-funding rate α based on the insurers' premium incomes as an insurance in good years, and provides bailouts in bad years when the insurer is in default. As mentioned above, the underwriting result at time t is given by

$$UR_\alpha(t) = (1-\alpha)TP(t) - X(t) - e(t). \tag{12}$$

and this leads to the operating result:

$$OR_\alpha(t) = UR_\alpha(t) + r \cdot (RAC(t) + TP(t)) + RAC(t) - RAC(t+1). \tag{13}$$

If operating result $OR_\alpha(t)$ is positive, it will be used to pay the loan from the bank first. After the loan has been cleared, the insurance company can distribute dividend. In contrast, if operating result $OR_\alpha(t)$ is negative, the insurer may raise some loan or even ask for fund to pass the financial distress. The cumulative liability, $L_\alpha(t)$, at the end of year t consists of previous-year's liability $L(t-1)$, the increment of risk adjusted capital $RAC(t+1)-RAC(t)$, and the operating result $OR_\alpha(t)$. If $L_\alpha(t)$ is higher than $\rho * RAC(t+1)$, where ρ is the trigger ratio for bailout, NCIF offers the insurer the fund $FUND_\alpha(t)$ i.e. the amount of $L_\alpha(t)$ over $\rho \cdot RAC(t)$. They are estimated by

$$L_\alpha(t) = \min\left[\max\left[L_\alpha(t-1) + RAC(t+1) - RAC(t) - OR_\alpha(t),\ 0\right],\ \rho \cdot RAC(t)\right], \qquad (14)$$

and

$$FUND_\alpha(t) = \max\left[\max\left[L_\alpha(t-1) + RAC(t+1) - RAC(t) - OR_\alpha(t),\ 0\right] - \rho \cdot RAC(t),\ 0\right]. \qquad (15)$$

Considering the adjustment of possible liability, the adjusted operating result at the end of year t is

$$AOR_\alpha(t) = OR_\alpha(t) - \max\left[L_\alpha(t-1) - L_\alpha(t),\ 0\right]. \qquad (16)$$

After paying the liability and fund, the insurer computes the profit before taxes $PBT_\alpha(t)$ by its adjusted operating result $AOR_\alpha(t)$ subtracted by the cost of liability $cL(t)$ as

$$PBT_\alpha(t) = AOR_\alpha(t) - cL(t), \qquad (17)$$

where c is the interest rate for liability. If the profit before taxes is negative, the authority provides a tax shield with rate γ to encourage the NatCat market by increasing future capital. The cumulative deferred taxes, $DTAX(t)$, in the year t could be calculated as

$$DTAX_\alpha(t) = \max\left[DTAX_\alpha(t-1) - \gamma \cdot PBT_\alpha(t),\ 0\right]. \qquad (18)$$

If the market is good and profit before taxes is positive, the taxable profit before taxes should be used to reimburse the cumulative deferred taxes happened at year t-1 ($DTAX_\alpha(t)$), and the rest will be taxed with a tax rate (τ). As a result, we could simply get the tax at year t, $TAX_\alpha(t)$, as equation (19) :

$$TAX_\alpha(t) = \tau \max\left[PBT_\alpha(t) - DTAX_\alpha(t-1),\ 0\right]. \qquad (19)$$

Therefore, the profit after taxes

$$PAT_\alpha(t) = PBT_\alpha(t) - TAX_\alpha(t). \qquad (20)$$

From the perspective of NCIFs, they receive payment from the insurer every year and need to pay the insurers some funds in the bad years. The capital changes of the NCIF at the end of year t is

$$AFUND_{\alpha}(t) = \sum_{i=1}^{t} (\alpha \cdot TP(i) - FUND_{\alpha}(i)) \cdot (1+r)^{t-i}. \qquad (21)$$

The related symbol in equation (21) are explained in previous equations.

2.5 The accounting procedure for insurers and NCIF with loan financing

The first program might potentially cause the insurer higher financial loading in good years and force the NCIF to set aside a huge capital if more insurers are difficult in bad years. The second program is much easier to both insurers and NCIF, assuming that NCIF provides some bailouts to the insurers suffering from financial distress in bad years. Its main difference from the first program is that the insurers are required to pay back the bailouts in good years. The insurers only need to pay NCIF a little annual premium rate (*l)* which may be close to risk-free rate because the principle of the loans are expected to return in the future.

As mentioned above, the underwriting result of an insurer at time t is given by

$$UR_l(t) = TP(t) - X(t) - e(t), \qquad (22)$$

and this leads to the operating result:

$$OR_l(t) = UR_l(t) + r \cdot (RAC(t) + TP(t)) + RAC(t) - RAC(t+1) - l \cdot AFUND_l(t-1) \qquad (23)$$

If operating result is negative, the insurer may raise some loans or even ask for funds to pass the financial distress. The cumulative liability $L_l(t)$ at the end of year t consists of the previous-year's liability $L_l(t-1)$) the increment of adjusted-risk capital $RAC(t+1) - RAC(t)$ and the operating result $OR_l(t)$. If $L_l(t)$ is higher than the hurdle point of bailout ($\rho * RAC(t)$), the NCIF would offer some funds to the insurer for reducing the liability to $RAC(t)$ level. All of the deduction could be modelled as

$$L_l(t) = \min\left[\max\left[L_l(t-1) + RAC(t+1) - RAC(t) - OR_l(t),\ 0\right],\ \rho \cdot RAC(t)\right], \qquad (24)$$

and

$$FUND_l(t) = \max\left[\max\left[L_l(t-1) + RAC(t+1) - RAC(t) - OR_l(t),\ 0\right] - \rho \cdot RAC(t),\ 0\right]. \tag{25}$$

On the other hand, if operating result is positive, it will be used to pay the loan borrowed from bank first and then recoup to the loan from NCIF. The insurance company could distribute dividend until the bailouts, including the loan and funds from the outside, which have to be paid back completely. Then, the rest of operating result, called adjusted operating result $AOR_l(t)$, at the end of year t could be estimated by

$$AOR_l(t) = \max\left[\max\left[OR_l(t) - \max\left[L_l(t-1) - L_l(t),\ 0\right],\ 0\right] - (1+l) \cdot AFUND_l(t-1),\ 0\right]. \tag{26}$$

The cumulative fund $AFUND_l(t)$ at the end of year t is highly associated with the market condition. Technically, it depends on the previous-year's cumulative fund ($AFUND(t-1)$) plus the possible fund of bailout $FUND_l(t)$ minus the possible amount of payback. It could be estimated by the following equation:

$$AFUND_l(t) = \max\left[AFUND_l(t-1) + FUND_l(t) - \max\left[OR_l(t) - \max\left[L_l(t-1) - L_l(t),\ 0\right],\ 0\right],\ 0\right]. \tag{27}$$

After paying the liability and fund, the insurer computes the profit before taxes $PBT_l(t)$ by using adjusted operating result $AOR_l(t)$ subtracting the cost of liability $cL_l(t)$, which is expressed by:

$$PBT_l(t) = AOR_l(t) - cL_l(t), \tag{28}$$

where c is the loan rate of liability. If the profit before taxes $PBT_l(t)$ is negative, the authority would provide a deferred rate γ to encourage the NatCat market by increasing future capital. After sequent NatCat events have occurred, the cumulative deferred taxes $DTAX(t)$ at year t is calculated by

$$DTAX_l(t) = \max\left[DTAX_l(t-1) - \gamma \cdot PBT_l(t),\ 0\right]. \tag{29}$$

If the profit before taxes $PBT_l(t)$ is positive, the cumulative deferred taxes $DTAX_l(t-1)$ should be excluded from taxable profit $PBT_l(t)$. Then, the taxes $TAX_l(t)$ with the rate, τ, could be calculated as

$$TAX_l(t) = \tau \max\left[PBT_l(t) - DTAX_l(t-1),\ 0\right]. \tag{30}$$

Finally, profit after taxes could be obtained as shown in equation (31).

$$PAT_l(t) = PBT_l(t) - TAX_l(t). \quad (31)$$

From the perspective of NCIFs, they receive premium from the insurer every year and pay the insurer funds during bad years. The capital changes of the NCIF at the end of year t is

$$CFUND_l(t) = (\sum_{i=1}^{t-1} l \cdot AFUND_l(i) \cdot (1+r)^{t-i}) - AFUND_l(t) \quad (32)$$

2.6 The accounting procedure for insurers and NCIF with equity financing

The last program gives insurers more flexibility and less short-term financial pressure. Insurers suffering from financial distress in bad years could exchange newly-issued shares for NCIF's bailouts. NCIF might sell insurers' equity in good years. In this circumstance, the insurer pays nothing to NCIF during the exchange. As mentioned in equation (4), the underwriting result at time t is given by

$$UR_e(t) = TP(t) - X(t) - e(t) \quad (33)$$

And similarly the operating result is

$$OR_e(t) = UR_e(t) + r \cdot (RAC(t) + TP(t)) + RAC(t) - RAC(t+1). \quad (34)$$

If operating result is negative, the insurer might ask for some funds from NCIF to get through the financial distress. The cumulative liability $L_e(t)$ at the end of year t consists of the previous-year's liability $L_e(t-1)$, the increment of adjusted-risk capital $RAC(t+1) - RAC(t)$ and the operating result $OR_e(t)$. If cumulative liability $L_e(t)$ is higher than the hurdle of bailout ($\rho \cdot RAC(t)$) with a trigger ρ, the NCIF offers some funds to the insurer to reduce the liability to $\rho \cdot RAC(t)$ level. All of the deductions could be modelled as shown in equation (35)

$$L_e(t) = \min\left[\max\left[L_e(t-1) + RAC(t+1) - RAC(t) - OR_e(t),\ 0\right],\ \rho \cdot RAC(t)\right], \quad (35)$$

where

$$FUND_e(t) = \max\left[\max\left[L_e(t-1) + RAC(t+1) - RAC(t) - OR_e(t),\ 0\right] - \rho \cdot RAC(t),\ 0\right]. \quad (36)$$

Considering the adjustment of possible liability, the adjusted operating result $AOR_e(t)$ at the end of year

t is

$$AOR_e(t) = OR_e(t) - \max\left[L_e(t-1) - L_e(t),\ 0\right]. \tag{37}$$

After paying the liability and fund, we can calculate the insurer's profit before taxes $PBT_e(t)$ by subtracting the cost of liability $cL_e(t)$ from adjusted operating result $AOR_e(t)$.

$$PBT_e(t) = AOR_e(t) - cL_e(t), \tag{38}$$

where c is the loan rate for liability. If the profit before taxes is negative, the authority provides a tax shield with rate γ to encourage the NatCat market by increasing future capital. The cumulative deferred taxes $DTAX(t)$ at year t could be calculated as

$$DTAX_e(t) = \max\left[DTAX_e(t-1) - \gamma \cdot PBT_e(t),\ 0\right]. \tag{39}$$

Oppositely, if the profit before taxes is positive, the cumulative deferred taxes $DTAX_e(t)$ needs to be excluded from the taxable profit $PBT_e(t)$. Then, the taxes could be computed as shown in equation (40)

$$TAX_e(t) = \tau \max\left[PBT_e(t) - DTAX_e(t-1),\ 0\right]. \tag{40}$$

Thus, we can get insurer's profit after taxes as:

$$PAT_e(t) = PBT_e(t) - TAX_e(t). \tag{41}$$

From the perspective of NCIFs, they provide bailouts to the insurers and receive their newly issued equities in bad years. Assume that an insurance company has n_0 shares outstanding at year 0 and issues new equities in exchange for bailout during the period from year 0 to year t. The number of new shares at year t depends on the ratio of bailout funds to $RAC(t)$ multiplied by a predetermined adjustment parameter (ω). For simplifying the model, the assets of insurance company, $A(t)$, is only assumed to be ready for writing policies at year t. i.e. $A(t)$ equals $RAC(t)$. We can get insurer's equity, denoted $E(t)$ at year t, using $A(t)$ to subtract the liability, $L(t)$. The theoretical share price $S(t)$ at year t could be estimated by the equity divided by the outstanding shares as exhibited in Equation (42).

$$S(t) = \max\left[\frac{A(t) - L(t)}{n_0 + n(t)},\ 0\right], \tag{42}$$

where $n(t)$ is the number of shares of accounted equities owned by NICF at the end of year *t*. When the insurer pays off his liability completely, the NCIF will sell the insurer's shares held by NCIF because of the good market price, and gather the income for the next bailout. Thus, $n(t)$ is estimated by the following equation:

$$n(t) = \left(n(t-1) + \omega \cdot \frac{FUND(t)}{RAC(t)} \cdot n_0\right) \cdot I_{[L(t)>0]} \tag{43}$$

where $I_{[L(t)>0]}$ is an indicator of $L(t)$.

On the other hand, the cumulative amount of bailout plus income $AFUND(t)$ of NCIF at the end of year *t* is estimated by

$$AFUND_e(t) = AFUND_e(t-1) + FUND_e(t) - S(t) \cdot n(t-1) \cdot I_{[L(t)>0]}. \tag{44}$$

The capital of NCIF decreases as $AFUND_e(t)$ becomes positive, meaning that NCIF pays bailout more than they receive from selling insurer's equity.

3. Simulation results

3.1 Data

In this section, we calibrate the loss model to carry out the numerical experiment. In this study, 45 observations of insured losses caused by natural disasters worldwide from 1970 to 2014 (in billion U.S. dollars) are collected from the journal "Sigma" issued by Swiss Re. The values are adjusted based on the consumer price index of the United States in 2014. Fig. 1 presents the time pattern of global insured losses. Obviously, there is an upward trend with irregular cycle, and it reached its historical high in 2005 and 2011, corresponding to Hurricane Katrina in 2005 and Japanese earthquake in 2011. Table 1 displays the descriptive statistics of insured losses. The loss sequence varies from 0.9631 in 1971 to 124.0216 in 2011, and its mean (volatility) is 22.7071 (27.0754). From the third and the fourth moments, the sample distribution is far from normal distribution. The Jarque-Bera test shows similar results, rejecting the null assumption of normality. In addition, the augmented Dickey-Fuller test indicates that the loss series is stationary. Figure 2 exhibits the sample autocorrelation of the loss sequence with 95% conference interval. It also shows that the sample autocorrelation at lag 1 (0.5047) is significant, meaning the natural disaster losses is serial dependent.

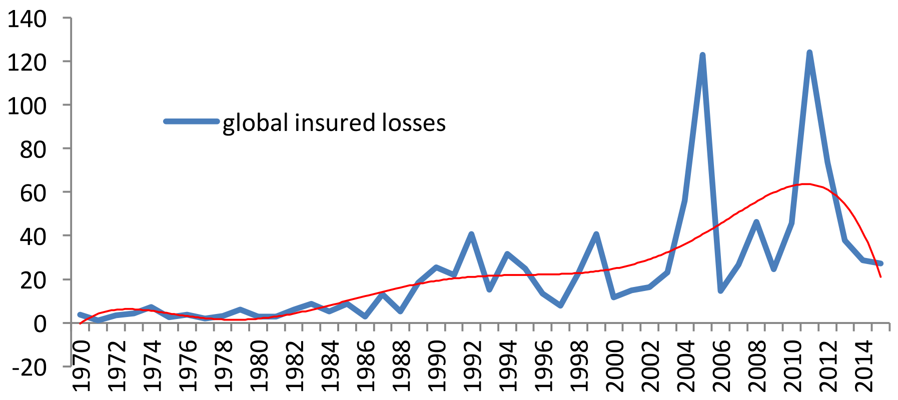

Fig. 1 The trend of global insured losses from 1970 to 2014

Table 1. Descriptive statistics of global insured losses from 1970 to 2014 in billion USD.

	N	Mean	Median	S.D.	Min	Max	Skew	Kurt.	J-B (*p-value*)	ADF (*p-value*)
insured losses	45	22.71	14.61	27.08	0.96	124.02	2.47	6.85	109.8 (0.001)	-2.63 (0.0098)

Notes: 1. Source: Sigma from Swiss Re 1970 to 2014 in 2014 prices.
2. This table shows the descriptive statistics for the insured losses. The number of observations (*N*), the mean, the median, the standard deviation (S.D.), the minimum (Min), the maximum (Max), the coefficients of skewness (Skew) and Kurtosis (Kurt) are reported. J-B denotes the test statistic for the Jarque-Bera normality test, which has a chi-square distribution with two degrees of freedom. ADF represents Augmented Dickey-Fuller test with the null hypothesis of a unit root.

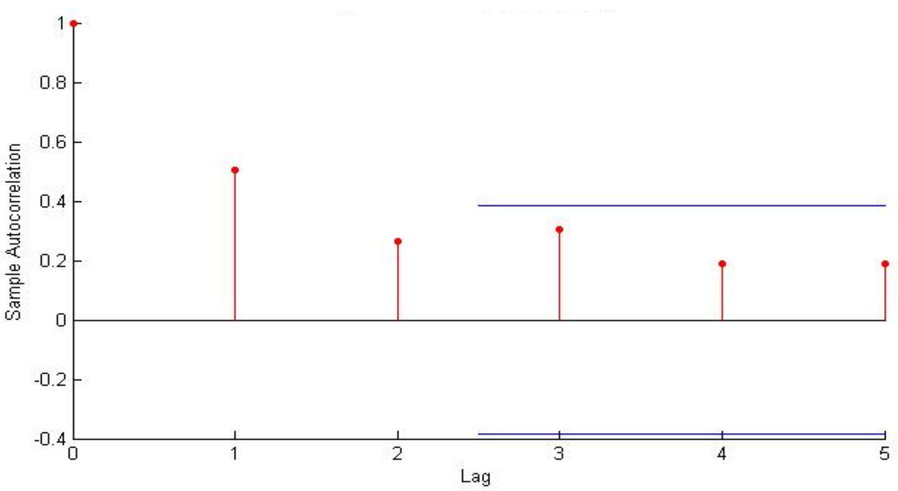

Fig. 2 The autocorrelation plot for the historical loss severities from 1970 to 2014

The results of the natural disaster loss distribution fitting are exhibited in Table 2, adopting maximum likelihood estimation to fit several well-known distributions such as lognormal (Burnecki et al. 2000), gamma (Jaimungal and Wang 2006), Weibull (Chernobai et al. 2006), generalized Pareto (Powers et al. 2012), generalized extreme value distributions (Abdessalem and Ohnishi 2013), exponential (Christensen 1999) and CIR (Cox et al. 1985) model. It shows that the CIR mode[3] fits the insured losses better than other distributions. It derives the largest log-likelihood function value (-3.7688) and the smallest AIC of 13.5376 and BIC of 18.9576 among them compared with the ones from others. The Monte Carlo estimate is based on 100,000 independent replicates calculated by the loss model over a period of 30 years. Referring the setup of Dacorogna, Albrecher, Moller and Sahiti (2013) and Wu (2015), the initial setting of the parameters in the simulation are defined in Table 3.

Table 2. The fitting results of distributions on global insured losses from 1970 to 2014 in billion USD.

distribution parameter estimated	lognormal	Gamma	generalized Pareto	generalized extreme value	Weibull	exponential	CIR
Scale	2.5228	23.4974	18.5369	1.7218	21.9530	22.6971	
Shape		0.9663	0.1860	0.2230	0.9365		
Location	1.1415			1.5015			
mean-reverting speed							0.4288
Mean							23.4616
instantaneous volatility rate							4.4144
Log-Likelihood	-183.335	-185.5031	-184.7545	-185.0352	-185.3406	-185.5007	-3.7688
AIC	370.6699	375.0062	373.509	376.0704	374.6811	373.0014	13.5376
BIC	374.2833	378.6196	377.1223	381.4904	378.2945	374.808	18.9576

Notes: 1. AIC denotes Akaike Information Criterion for estimated model
2. BIC denotes Bayesian Information Criterion
3. CIR model denotes Cox-Ingersoll-Ross mean-reverting square root models

3 More details of this model are presented in Appendix

Table 3 Standard set of parameters

Standard parameters	
the initial value of claim	25.3012 billion USD
risk quantile, θ	0.99
risk-free rate, r	2%
capital raising cost, c	3%
shareholder's required return, κ	15%
expense, e	1% of expected loss
tax rate, τ	25%
tax shield rate, γ	25%
maturity year, T	30
simulation times	100,000
a trigger ratio for bailout, ρ	0.05
a pre-funding rate, α	0.8
an annual premium for loan financing, l	0.02
a predetermined ratio for equity financing, ω	1.5

Notes: The parameters of a simulation base are defined in Table 3, which refers to the setup of Dacorogna, Albrecher, Moller, and Sahiti (2013) and Wu (2015). The claim in previous year is estimated in Appendix.

After obtaining the best fitting distribution, it seems necessary to demonstrate that the insurers will not default and NCIF could achieve self-finance under the three proposed financing programs.

3.2 The impact of taxes in basic accounting procedure on insurer's default rate

The first step to prove that the proposed programs is effective is to show that the insurer will not default under the best-fitting distribution (CIR process) with all tax rate. We assume that the insurer's tax shield rate is equal to its tax rate. It is known that tax reduces insurer's profit but tax shield discounts insurer's losses. The result leads to the simultaneous decrement of financial performance and default rate. After obtaining the parameters of the CIR model discussed in section 3.1, the simulation is implemented 100,000 times to every possible tax rate. As discussed in section 3.1, if operational performance is less than negative risk-adjusted capital, the insurer defaults. Figure 3 shows the pattern between expected default rate and tax rates. Overall, the insurer's default rate stays low, and the insurer's default rate is negatively associated with its tax rate, i.e. the higher tax rate produces a lower default rate. In other words, higher tax rate brings more effects from tax shield reducing the likelihood of default. Therefore, the authority might give consideration to tax rate setting and insurer's financial performance. A proper tax rate could help insurers to reduce their default rate.

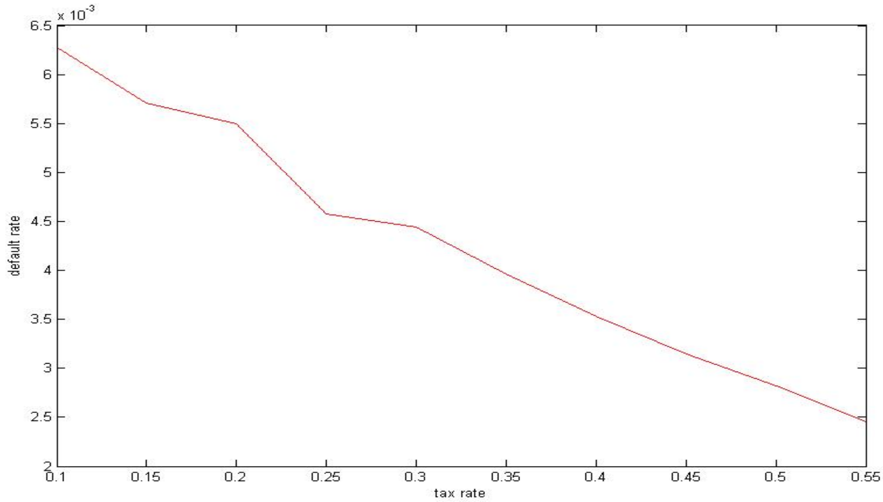

Fig. 3 The expected default rate under different tax rates

3.3 The impact of pre-funding rate on the self-finance of NCIF

After proving that the insurers have low default probability, this section shows that NCIF can achieve self-finance within the pre-funding program mentioned in section 2.4. In the pre-funding program, NCIF's financial revenue comes from the pre-funding rate multiplied by the premium income of insurers, and the expenditure is the bailout fund in bad years. The capital change of NCIF is shown in equation (32). Figure 4 shows the pattern of NCIF's financial revenue and expenditure to various pre-funding rates (α). In general, NCIF's revenue and expenditure is positively correlated with its pre-funding rate. The mean of financial revenue and expenditure (in red) exhibits a positive trend from zero, and a higher pre-funding rate contributes more financial revenue and expenditure. This implies that if NCIF charges insurers a higher pre-funding rate then NCIF can generate more funds to help the insurers pass the financial distress. However, unreasonably high pre-funding rates might reduce insurers' willingness to join this program. In summary, NCIF can achieve its financial self-sufficiency by choosing an appropriate pre-funding rate.

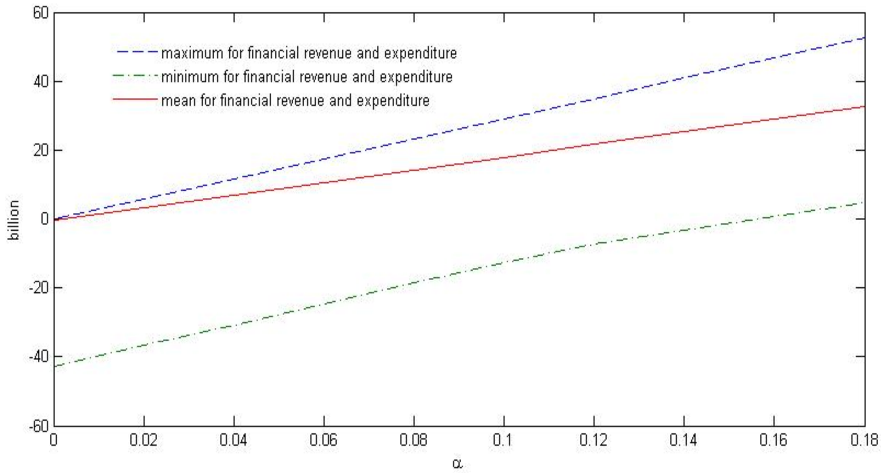

Fig. 4 The maximum, minimum and mean for financial revenue and expenditure of NCIF under different pre-funding rates (α)

3.4 The impact of loan-financing rate on the self-finance of NCIF

The objective of this section is to prove that a loan-finance program could lead NCIF to achieve self-sufficiency in their financial operation as shown in equation (32) in section 2.5. The loan-financing program requires insurers receiving bailout to pay back the loan plus certain interest when they have positive profit before tax. According to the CIR loss model, the insurers will eventually clean off the loan because NCIF helps them to avoid default. Figure 5 describes the pattern of financial revenue and expenditure to various loan-financing rate. Clearly, it shows our expectations that the sum of financial revenue and expenditure stably increases as the loan-financing rate increases. In other words, the mean value of NCIF's capital change (interest income minus bailout amount) is larger than zero if it asks the insurer to pay a lower loan rate (around 0 to 2%). And in most cases NCIF can operate this program well if the loan rate is higher. The mean value of the sum nearly overlaps the maximum value because most loans are cleared off within 30 years. The minimum value of the sum is always negative and it approaches zero from -8 billion. This phenomenon is associated with some fairly-new loans and they have not been paid off within a 30-year simulation period. It is clear that higher loan rates increase insurer's interest payment (or NCIF's financial revenue) and then decreases the financial performance of insurance companies. Considering NCIF's role in the market as a non-profit organization, it would be better to find a proper loan rate to balance NCIF's financial revenue and bailout expenditure by adjusting the rate based on changes of insured losses.

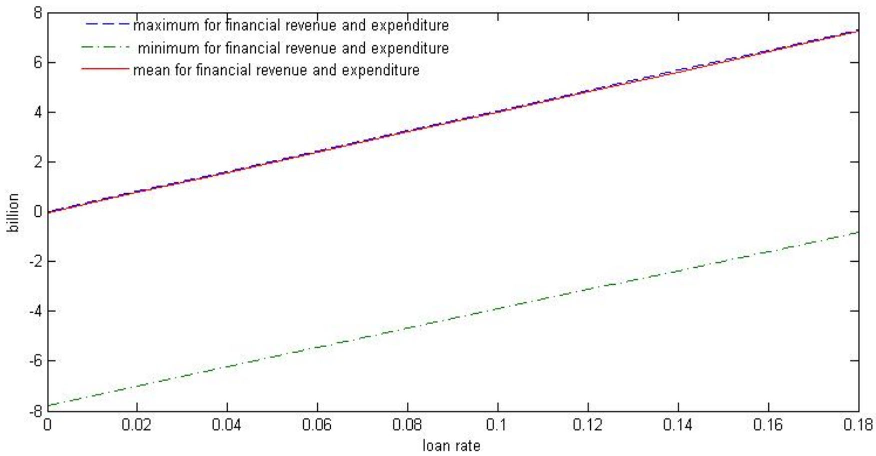

Fig. 5 The maximum, minimum and mean for financial revenue and expenditure of NCIF under different loan-financing rates

3.5 The impact of equity-financing ratio on the self-financing of NCIF

The last one program is equity-financing, which helps an insurer exchange NCIF's bailout funds with the insurer's newly issued equities. NCIF's cumulative bailout plus its income is shown in equation (44) in section 2.6. Intuitionally, insurer's equity will be underpriced when the insurer is in certain distress caused by NatCat. Figure 6 shows that the pattern of financial revenue and expenditure to various equity-financing ratios, ω. The sequence of mean value is around zero, indicating, on average, NCIF can achieve self-sufficiency in the long term and NCIF would not take any advantage from the insurers.

On the other hand, both the patterns of maximum and the minimum value of NCIF's financial revenue and expenditure are negatively correlated with the equity-financing ratio. It is obscure and unexpected. One possible explanation to this phenomenon is the dilution effect. More specifically, even if NCIF could obtain more insurer's equity in the case of higher equity-financing ratio, the newly issued shares might dilute insurer's financial performance (for example, earnings per share; EPS) and further cause a discount in share price. Thus, higher equity-financing ratio will reduce NCIF's revenue from selling insurer's shares even though the insurer's profit before tax becomes positive. Overall, equity-financing program is worthy to adopt since it offers less financial pressure to the insurers while allowing NCIF to achieve its self-sufficiency in the long term.

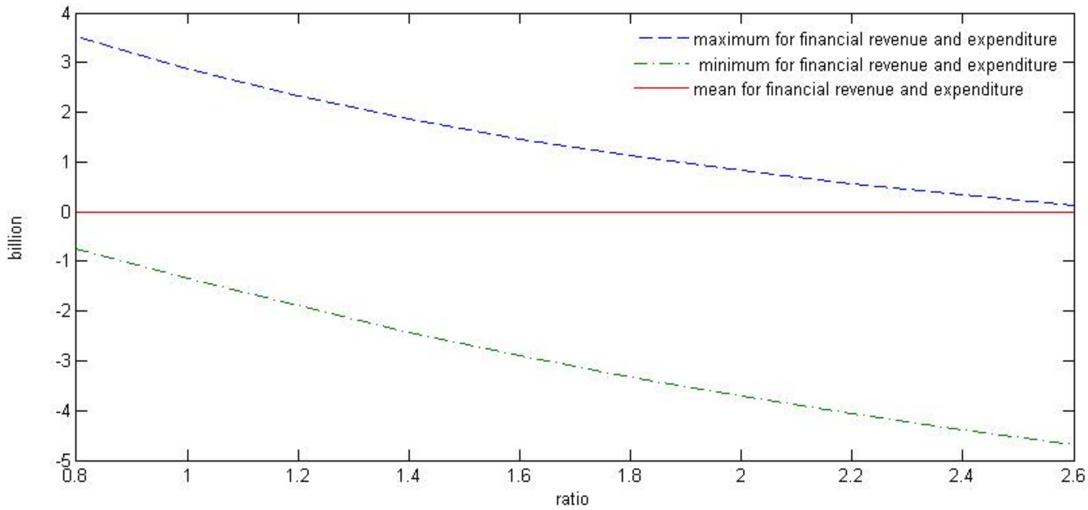

Fig. 6 The maximum, minimum and mean for financial revenue and expenditure of NCIF under different equity-financing ratios

4. Conclusion

Natural catastrophes have become more frequent over the past three decades. Most countries established a reasonable insurance mechanism to respond to the consequences of NatCats. A natural catastrophe insurance fund (NCIF) can strengthen insurance schemes by promoting the affordability and availability of insurance and spreading NatCat risk among involved parties (insureds, insurers and reinsurers). In addition, it also ensures the capacity at a reasonable price. The financial self-sufficiency of NCIF can make sure that this mechanism works in the long term and reduce government's fiscal load.

This study shows the feasibility of financial self-sufficiency if NCIF bailout the insurer by three following programs: 1) the pre-funding means that NCIF receives insurer's some fund in good years and offers bailout to insurers in bad years, 2) the loan-financing means that NCIF requires the insurer to pay back the bailout loan, and 3) the equity-financing allows the insurers exchange bailout for his under-priced equity during the period of distress. We calibrate the loss model using global insured losses, and design corresponding accounting procedures for insurer and NCIF with different bailout programs. For each Monte Carlo simulation, the cash flows are estimated based on these accounting procedures over a period of 30 years under such loss model and rational parameter assumption. The numerical results are calculated using 100,000 independent replicates. The numerical analysis implies that three bailout programs can balance both the financial revenue and bailout expenditure of NCIF.

Appendix

The CIR model is in the form of $dL(t) = a(b - L(t))dt + \sigma_L \sqrt{L(t)}dW(t)$, where $L(t)$ stands for insured losses; a, b, and σ_L denote the mean-reverting speed, mean, and instantaneous volatility rate, respectively. W follows the Wiener process. Furthermore, it could be illustrated that 2c$L(t)$ follows a non-central chi-square χ^2 distribution with $2q$ + 2 degrees of freedom and non-centrality parameter 2u, where $c = \frac{2a}{\sigma_L(1 - e^{-a})}$, $u = cr(t-1)e^{-a}$, $q = \frac{2ab}{\sigma_L} - 1$. According to the parameters estimated in Table 2, without loss of generality, we estimate the initial value of claim 2c$L(0)$ given a = 0.4288, b = 23.4616, σ_L = 4.4144, and $L(0)$ = 22.7071 (the mean of 45 observations).

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